

# A hybrid method to analyse the performance of falling film absorbers

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(Received 15 July 1991 and in final form 3 April 1992)

**Abstract**—The equations to describe the behaviour of a cooled falling film absorber with laminar flow in the absorbent stream are presented and solved numerically by using a novel hybrid method of solution which combines an analytical solution with the finite difference method. Theoretical performance data were obtained in terms of absorber effectiveness, number of transfer units and coolant capacity ratio. These data showed that, for given entry conditions, absorber performance can be improved by increasing the coolant flow rate and/or the contact area for heat and mass transfer. Indications of how absorber performance can be optimized are given.

## INTRODUCTION

ABSORPTION is common in engineering processes. It is used in absorption refrigeration and heat pump systems where the absorber is the means by which vapour is extracted from the evaporator. It replaces the compressor which is used in the alternative vapour compression systems [1]. In some applications the absorption process can be taken to be isothermal and a description of the process requires only consideration of the mass transfer phenomena [2, 3]; however, in other applications including the refrigeration and heat pump cycle, which is emphasized in this paper, there is significant evolution of heat during absorption and the description of the process requires the heat and mass transfer effects to be considered simultaneously [4-6]. In the refrigeration process the absorber will maximize the refrigeration capacity by maximizing the rate at which vapour is extracted from the evaporator. This is achieved by making the change in concentration of the absorbent as it passes through the absorber as large as possible. To meet this aim it is necessary to cool the absorbent during the absorption process. This point is best illustrated on a diagram such as Fig. 1 where the coordinates are absorbent temperature and concentration at a fixed vapour pressure. The equilibrium line is shown and any absorbent condition above this line would be in a state of desorption and any condition below would be in a state of absorption. State 'I' is a typical state entering from a regenerator. The path 'I' to 'A' is that of an adiabatic change with infinite contact area between the absorbent and the vapour. The path 'I' to 'B' is the change for an infinite contact area where the absorbent, in contra-flow, is cooled to the entering coolant temperature. The corresponding leaving state for a finite contact area is 'D'. It can be clearly seen from this diagram that to achieve a large change in concentration, the absorbent must be cooled and the con-

tact areas between the vapour and absorbent and the coolant and the absorbent must be as large as possible. To meet this requirement various arrangements can be used, but the arrangement finding increasing favour, because it has been shown to be effective [7, 8], is the falling film type. This type of system has been investigated by Nakoryakov and Griger'eva [4-6] and by Grossman [9] where the cooling surface has been either isothermal or adiabatic. In this paper we extend the treatment to include the

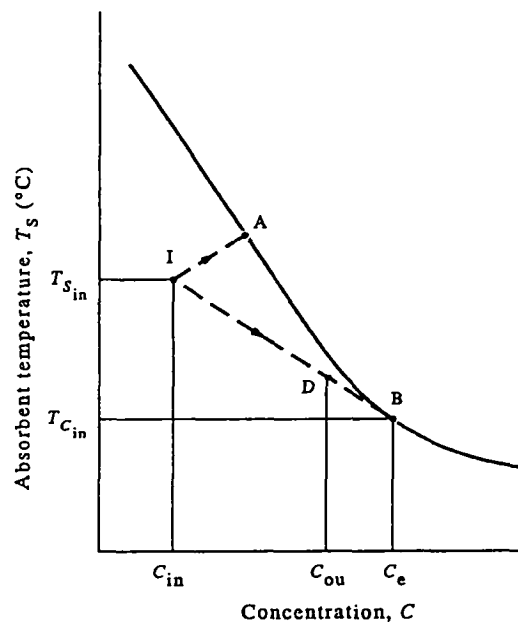


Fig. 1. Typical change of temperature with concentration of the absorbent as it passes through the absorber. The broken lines show the paths and the full line is the equilibrium line.



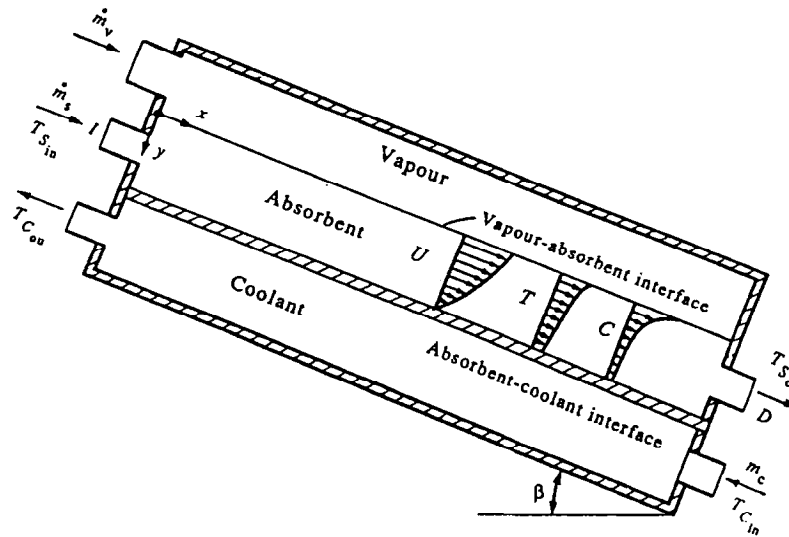


FIG. 2. Schematic of the falling film absorber showing the regions occupied by the vapour, the absorbent and the cooling fluid which is shown in contra-flow.

At the interface between the absorbent and the coolant the boundary conditions are

$$\frac{\partial C}{\partial y} = 0 \tag{4}$$

$$\frac{\partial T_s}{\partial y} = \frac{h_t}{\lambda} (T_w - T_c) \tag{5}$$

where the overall heat transfer coefficient  $h_t$  takes account of the resistance in the coolant stream and in the separating bulkhead. The coolant stream resistance was obtained from the Dittus-Boelter equation found in ref. [13]. The coolant flow was assumed, therefore, to be fully developed.

At the absorbent vapour interface the boundary conditions are

$$\frac{\partial T_s}{\partial y} = \frac{\dot{m}_v}{\lambda} \delta h \tag{6}$$

where

$$\delta h = h - [h + (1 - C) \partial h / \partial C] \tag{7}$$

and

$$\dot{m}_v = \frac{\rho D}{1 - C} \frac{\partial C}{\partial y} \tag{8}$$

It is assumed that the absorbent concentration and temperature are in equilibrium at the interface for the prevailing vapour pressure, i.e.

$$T_{si} = f(C_i) \tag{9a}$$

where  $C_i$  is the equilibrium concentration at  $T_{si}$ .

The actual equilibrium relationship between temperature and concentration for the particular vapour pressure will be known for the vapour/absorbent combination and the equilibrium data for LiBr/H<sub>2</sub>O are

plotted in Fig. 3. This figure shows that, for the range of conditions obtained in absorption refrigerators, the relationship between temperature and concentration at equilibrium is approximately linear, i.e.

$$T_{si} = a + bC_i \tag{9b}$$

Now if the accurate equation is used then an implicit equation has to be solved at the boundary, whereas if the equation is approximated to linear then the equation becomes explicit and much easier to handle. In this paper both the actual and the linearized data have been used in the solution to test the reasonableness of the linear approximation. The heat of absorption depends on both the temperature and concentration at the interface but, with very little error, it can be set constant and this is the condition assumed here. It is further assumed that enthalpy transfer as a result of a temperature difference between the vapour and the interface is negligible. It will be clear that, at entry to the absorber, there is likely to be a sudden change in absorbent conditions at the interface to attain equilibrium. A point of singularity will, therefore, exist which can lead to problems with the solution.

By putting

$$X = \frac{x\alpha}{\dot{u}\Delta^2}; \quad Y = \frac{y}{\Delta}; \quad \gamma = \frac{C - C_o}{C_e - C_o};$$

$$\theta_s = \frac{T_s - T_o}{T_e - T_o} \quad \text{and} \quad L_c = \frac{D}{\alpha}$$

where  $T_o$  and  $C_o$  are the absorbent conditions at the absorber inlet,  $C_e$  is the equilibrium concentration at  $T_o$  and  $T_e$  is the equilibrium temperature at  $C_o$ , equations (1)–(3) can be non-dimensionalized into the form

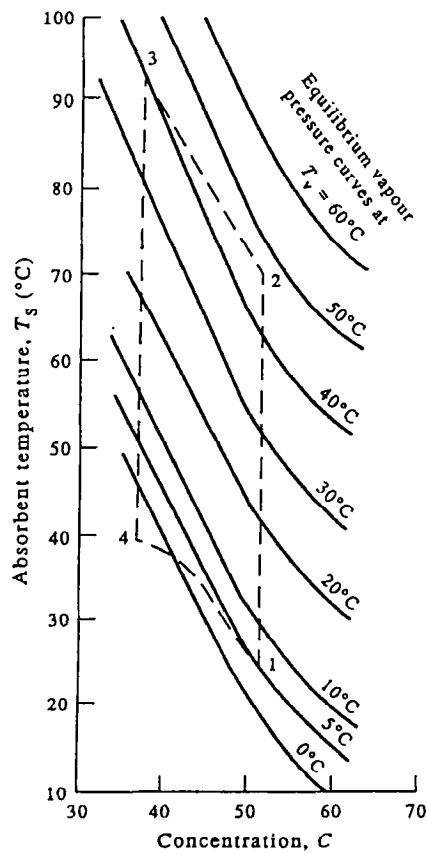


FIG. 3. The relationship between temperature and concentration, at equilibrium, for a LiBr/H<sub>2</sub>O solution at various vapour pressures. The cycle 1-2-3-4 is that typical for an absorption refrigeration system.

$$U = \frac{3}{2}(1 - Y^2) \quad (10)$$

$$U \frac{\partial \gamma}{\partial X} = Le \frac{\partial^2 \gamma}{\partial Y^2} \quad (11)$$

$$U \frac{\partial \theta_s}{\partial X} = \frac{\partial^2 \theta_s}{\partial Y^2} \quad (12)$$

At the coolant interface ( $Y = 1$ )

$$\frac{\partial \gamma}{\partial Y} = 0 \quad (13)$$

$$\frac{\partial \theta_s}{\partial Y} = Nu_c(\theta_w - \theta_c) \quad (14)$$

At the vapour interface ( $Y = 0$ )

$$\frac{\partial \theta_s}{\partial Y} = H \cdot \frac{\partial \gamma}{\partial Y} \quad (15)$$

where

$$H = \frac{\rho D}{(1 - C_o)\lambda} \cdot \frac{C_c - C_o}{T_c - T_o} \cdot \delta h \quad (16)$$

Interface equilibrium is governed by

$$\theta_{si} = f(\gamma_i) \quad (17a)$$

which is linearized into

$$\theta_{si} + \gamma_i = 1. \quad (17b)$$

## SOLUTION

The obvious means of solution was by way of the implicit finite difference method directly. However, it was found that because of the existence of the singularity and because for practical vapour/absorbent combinations the Lewis number is very small (for LiBr/H<sub>2</sub>O it is approximately 0.008), then to obtain acceptable accuracy, i.e. a mesh-independent solution, a very fine mesh size was required, particularly in the entry region. Consequently the solution time for an absorber of reasonable length was considerable. To reduce the solution time a hybrid method of solution has been adopted which combines an analytical solution with the implicit finite difference method. The method is fully described in ref. [14] and is based on an analytical solution described in ref. [5]. A brief description follows. It is known that the analytical solution is accurate near the entry region and so it was used to obtain the concentration and temperature profiles, as starting values for the finite difference method, at the maximum distance from the entry region where it is known to be still accurate. This distance is small compared with a practical absorber length but nevertheless far enough away from the singularity to avoid the problems with it which can, otherwise, only be partially overcome by utilizing very fine, finite difference meshes. Grossman [9] used this technique for the vapour/absorbent interface at entry but here it has been extended across the whole thickness of the absorbent film to include the cooling boundary also. The analytical solution is also accurate near the vapour/absorbent interface for any length of absorber, and so it has been used at the interface to evaluate the boundary conditions along the whole length. With this hybrid technique together with an algorithm to automatically optimize the mesh size (fully described in ref. [14]) a significant decrease in computational time was obtained to achieve the required accuracy. In Fig. 4 some results are compared. All the results in the figure were obtained using the analytical method in the entry region so they all have the same starting values up to  $X = 0.006$ . However, a and b used the direct finite difference method at the interface and c and d used the analytical method. It is noted that there is good agreement between the mesh size of  $DY = 1/80$  and  $DY = 1/20$  for the fully hybrid solution (curves c and d); the mesh size of  $DY = 1/320$  for the partial hybrid solution (curve b) approaches this accuracy but the partial hybrid solution with a mesh size of  $DY = 1/20$  (curve a) shows very poor correspondence. Typical comparative computing times between curve b and c were 150 and 50 min on the VAX CLUSTER. The com-

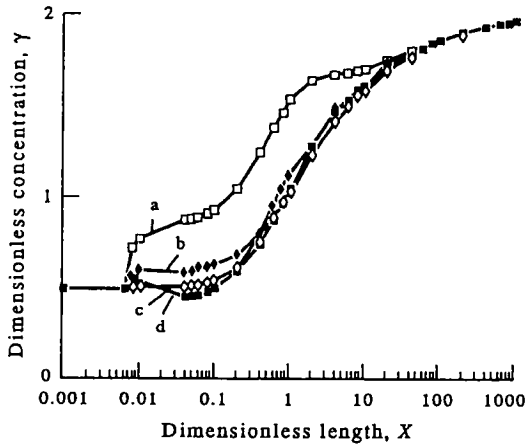


FIG. 4. Comparison between the variation of interface concentration with dimensionless length using different solution techniques.  $Le = 0.0001$ ,  $H = 0.01$ ,  $Nu_c = 10$  and  $R = 100$ : (a) finite difference method with  $DY = 1/20$ ; (b) finite difference method with  $DY = 1/320$ ; (c) hybrid method with  $DY = 1/80$ ; and, (d) hybrid method with  $DY = 1/20$ .

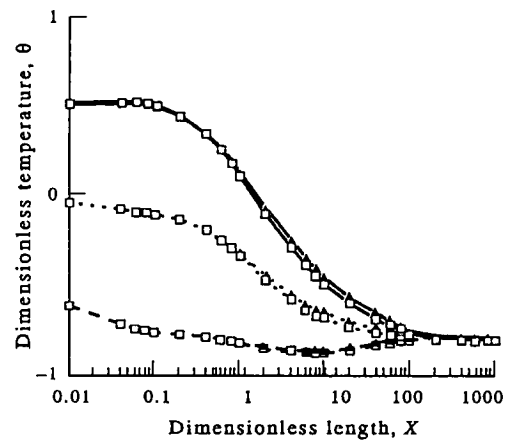


FIG. 6. Variation of temperature with length for a LiBr/H<sub>2</sub>O solution comparing the linear and non-linear boundary condition at the interface.  $\theta_s = 0$ ,  $\theta_c = -1.0$ ,  $R = 100$ ,  $Nu_c = 10$ ,  $Le = 0.0083$  and  $H = 0.0934$ : (a) and (b) interface states non-linear and linear; (c) and (d) mean states non-linear and linear; and (e) and (f) wall states non-linear and linear.

parative time to get the same accuracy will be considerably longer.

The justification for adopting the linearized equation at the interface was tested and a comparison between the actual condition (equation (17a)) and the linearized condition (equation (17b)) is made in Figs. 5 and 6 for conditions obtained in absorbers for refrigeration and heat pump systems using a LiBr/H<sub>2</sub>O combination. It is seen that for absorbers with  $X < 10$  there will be little error in adopting the

linear form of the equations. This conclusion supports the opinion of Grossman [9].

In Fig. 7 a comparison between results from the present method are compared with previous workers. The ordinate is the Sherwood number  $Sh$  where

$$Sh = \frac{\partial \gamma}{\partial Y} \Big|_{Y=1} \cdot \frac{1}{\gamma_i - \bar{\gamma}}$$

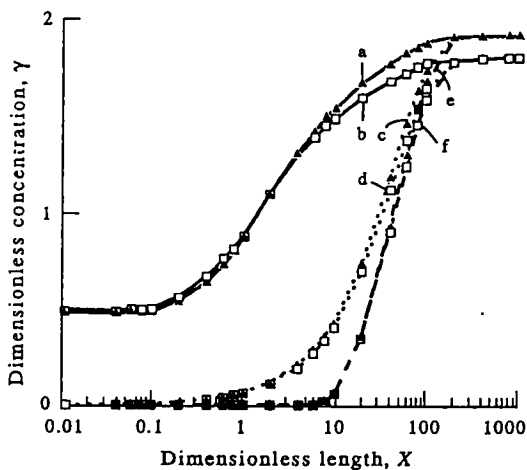


FIG. 5. Variation of concentration with length for a LiBr/H<sub>2</sub>O solution comparing the linear and non-linear boundary condition at the interface.  $\theta_s = 0$ ,  $\theta_c = -1.0$ ,  $R = 100$ ,  $Nu_c = 10$ ,  $Le = 0.0083$  and  $H = 0.0934$ : (a) and (b) interface states non-linear and linear; (c) and (d) mean states non-linear and linear; and (e) and (f) wall states non-linear and linear.

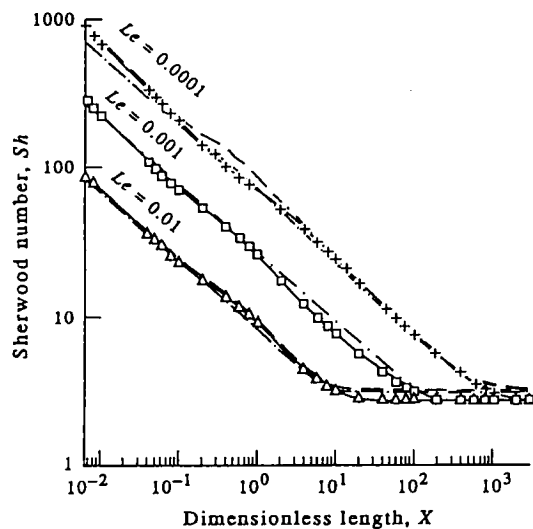


FIG. 7. Variation of Sherwood number with dimensionless length for various values of the Lewis number comparing the present study with previous workers.  $H = 0.01$ . Grossman's results (---), where  $\theta_s = \theta_c (= 0)$ . Results of Van der Wekken and Wassenaar (—), where  $\theta_s = 0$ ,  $\theta_c = -1.0$  and  $Nu_c = 10$ ; and the present work (—), with the same parameter values as Van der Wekken and Wassenaar.

All results are for isothermal conditions at the coolant interface. The results for Van der Wekken and Wasenaar [10] are for the same conditions as the present study, i.e.  $\theta_s = 0$  and  $\theta_c = -1.0$ . In the case of Grossman [9] the coolant and the absorbent are, in effect, at the same temperature, i.e. both  $\theta_s$  and  $\theta_c$  are zero. It is apparent that at large values of  $X$  there is better agreement between the previous workers than the present study. However, for the practical range of  $X$ , i.e.  $0.01 < X < 10$ , there is some disagreement between the previous workers and it is further noted that within this range, at any point, the results of the present study agrees, very well, with one or the other of the previous results. It is emphasized that the mesh size used to obtain the present results was such that the results were mesh size independent.

### RESULTS AND DISCUSSION

A convenient parameter for the presentation of results is the number of transfer units. This is, in effect, a measure of the relative size of an absorber. This parameter can take various forms but the form selected must be such that it can be quantified a priori. To this end the number of transfer units is defined here as

$$NTU = h_c A / (\dot{m} C_p)_c \quad (18)$$

The concept of effectiveness to describe the performance of heat exchangers is a useful one and so it has been adopted. For the case of an absorber we define its effectiveness  $E$  as the actual vapour flow rate  $\dot{m}_v$  divided by that vapour flow rate obtained from an ideal absorber where the absorbent leaves the absorber with an equilibrium concentration and at the same temperature as the entering cooling medium,  $\dot{m}_{mx}$ , i.e.

$$E = \dot{m}_v / \dot{m}_{mx} \quad (19)$$

Thus an effectiveness of unity implies an absorber with an infinite contact area.

The other parameter used, which in effect quantifies the cooling capacity of the coolant, is the capacity ratio  $R$  where

$$R = (\dot{m} C_p)_c / (\dot{m} C_p)_s \quad (20)$$

The values used in equation (20) are at the entry conditions for both fluid streams.

$NTU$  and  $R$  are related to  $Nu_c$  by the expression

$$NTU = Nu_c X / R \quad (21)$$

In practice, a contra-flow heat exchanger with no phase change will always be more effective than a parallel-flow device, whereas for an evaporator or condenser, where one of the fluid streams is at constant temperature, the relative flow direction is of no consequence. It is expected that the behaviour of an absorber would lie somewhere between the two. This

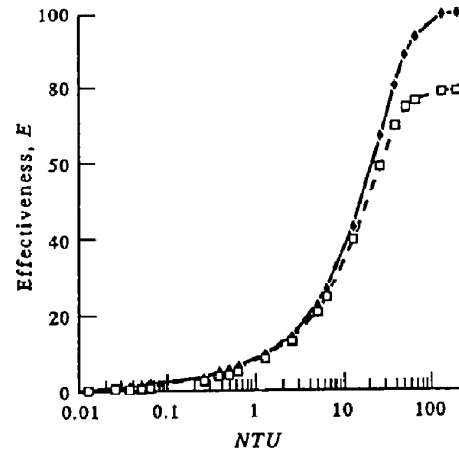


FIG. 8. Comparison between effectiveness for parallel- and contra-flow cooling. The full line is contra-flow and the broken line is parallel-flow.  $Le = 0.001$ ,  $H = 0.01$ ,  $R = 50$  and  $\theta_{c_{in}} = -1/3$ .

is confirmed in Fig. 8, where the contra-flow absorber is compared with the parallel-flow system. It is seen that contra-flow is more effective but the margin of difference, compared with a heat exchanger with no phase change, is reduced because of the heat of absorption.

Figure 9 shows the relationship between the number of transfer units and the effectiveness for a contra-flow system. It can be seen that in order to maximize  $E$  then  $Le$  should be as large as possible. Now the refrigerating capacity is proportional to  $E$  and, to a first approximation, proportional to the heat of absorption  $H$ . It is dependent on  $Le$  only by the dependence of  $Le$  on  $E$ . Thus Fig. 9 tells us that the refrigerating capacity will also increase with an

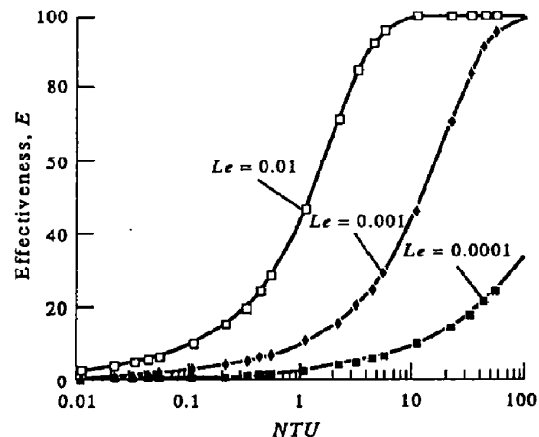


FIG. 9. The effect of Lewis number on the change of effectiveness with the number of transfer units.  $H = 0.01$ ,  $Nu_c = 1.0$ ,  $R = 100$ ,  $\theta_{c_{in}} = -8/3$  and  $\theta_{c_{ou}} = -1/3$ .

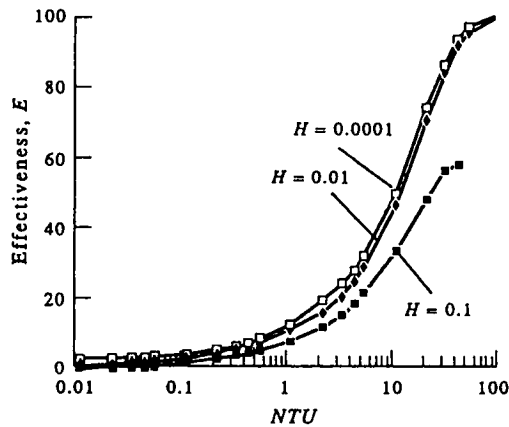


FIG. 10. The effect of heat of absorption on the change of effectiveness with the number of transfer units.  $Le = 0.01$ ,  $Nu_c = 1.0$ ,  $R = 100$ ,  $\theta_{\text{in}} = -8/3$  and  $\theta_{\text{coo}} = -1/3$ .

increase in  $Le$ . In Fig. 10 the corresponding effect of  $H$  is shown. Here it can be seen that decreasing  $H$  will increase  $E$ . However, with respect to refrigerating capacity, its direct dependence on  $H$  is more dominant and so, overall, the refrigerating capacity will increase with increasing  $H$ . The data for both figures were obtained for a capacity ratio of 100, with the cooling water leaving at  $\theta_w = -1/3$ . A few results were taken at other conditions but because the computing time was considerable these additional runs had to be limited. However, the additional data which were collected indicated that the effectiveness was not significantly affected by different entry conditions.

The absorbent pair most used for refrigeration applications is LiBr/H<sub>2</sub>O and the effectiveness, for this particular combination for a contra-flow system and how it is affected by both  $R$  and  $NTU$ , is shown in Fig. 11. The required values of  $R$  are much larger

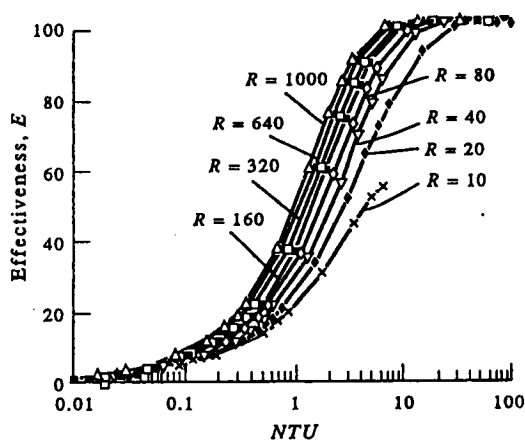


FIG. 11. The effect of heat capacity ratio on the change of effectiveness with the number of transfer units for a LiBr/H<sub>2</sub>O combination.  $T_{\text{sin}} = 25^\circ\text{C}$ ,  $T_{\text{coo}} = 35^\circ\text{C}$  and  $Nu_c$  is varied with  $R$ .

than that typical for heat exchangers because the coolant has to deal with the heat of absorption. The figure illustrates clearly that, for a given value of  $R$ , then after a certain value of  $NTU$ , there is a diminishing rate of increase of  $E$  on increasing  $NTU$ , e.g. by increasing the absorber contact area. For example, with  $R = 320$  and  $NTU = 3$  the effectiveness is 0.8 whereas increasing  $NTU$  to 30 (by, say, using an absorber of 10 times the size) would increase  $E$  by only a further 20% approximately. The effect of changing the absorber area can be clearly discerned from Fig. 11. However, it is not easy to see the effect of changing the coolant flow rate because the coolant flow rate is contained within  $NTU$  (directly by  $m_c$  and indirectly by  $h_c$ ), and also in  $R$ . Thus the effect of changing  $m_c$  is best illustrated by plotting  $E$  on a base of  $R$  and using the area ratio  $\bar{A}$  to indicate relative absorber size. The area ratio is defined by

$$\bar{A} = A/A_\infty \quad (22)$$

where  $A$  is the actual contact area and  $A_\infty$  is the contact area to attain 99.99% of equilibrium concentration, at exit, at the coolant entering temperature. Of course 100% cannot be used because this requires an infinite contact area. The actual figure, Fig. 12, shows that similar considerations apply as for the contact area, i.e. after a certain flow rate there is an ever decreasing return on increasing the coolant flow rate further. The conclusion from Fig. 12 is simpler, however. It implies that for all absorber areas a capacity ratio between 20 and 40 should be used.

It is apparent from Fig. 11 that the optimum value of  $NTU$  would depend upon the value of  $R$  and vice versa. To illustrate this point, consider a refrigerating system using LiBr/H<sub>2</sub>O with typical condensing and evaporating temperatures of 50 and 5°C, respectively, a boiling temperature of 110°C, a strong solution, in equilibrium, entering the absorber at 40°C and

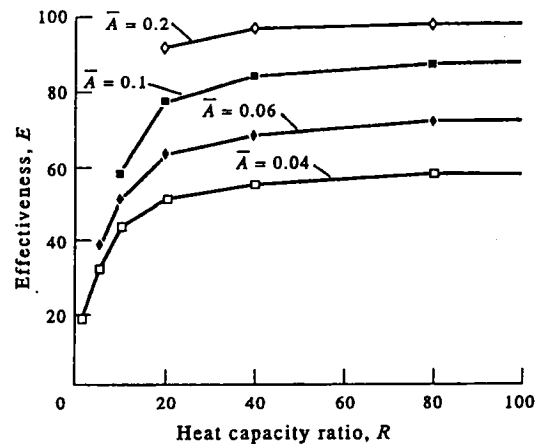


FIG. 12. The effect of area ratio on the change of effectiveness with heat capacity ratio for a LiBr/H<sub>2</sub>O combination.  $T_{\text{sin}} = 25^\circ\text{C}$ ,  $T_{\text{coo}} = 35^\circ\text{C}$  and  $Nu_c$  is varied with  $R$ .

cooling water entering the absorber at 20°C. If the absorber was 0.5 m wide then the corresponding solution flow into it would be 0.04 kg s<sup>-1</sup> and for an ideal absorber with an effectiveness of unity, the refrigerating capacity would be 43 kW. If the absorber was 3 m long and the capacity ratio was 20 then the calculated *NTU* is 0.78 and from Fig. 11 the corresponding effectiveness is 0.2. This results in an actual refrigerating capacity of 8.6 kW. This capacity could be increased by increasing the absorber contact area and/or increasing the water flow rate. If the area is doubled then the refrigerating capacity is increased significantly to 13.8 kW, whereas if the water flow rate is doubled then there is only a marginal increase in refrigerating capacity to 9.0 kW. In this case, therefore, increasing the area would probably be recommended. In other situations the converse could be true.

It is apparent from Fig. 11 that the same value of *E* and hence refrigerating capacity can be achieved by different combinations of *NTU* and *R*. The figures show these optimum combinations, for a given *E*, but the combination selected in practice would result from, in effect, comparing the relative cost of absorber contact area with the cost of supplying coolant flow rate. The data from Fig. 12 should make the selection process easier because they show that the reasonable range of *R* is restricted.

### CONCLUSION

It has been shown that an accurate and faster method of obtaining the performance of a cooled falling film absorber can be obtained using a hybrid method of solution which combines an analytical treatment in the regions where it is accurate, i.e. at the entry region and at the interface between the vapour and the absorbent stream, and a finite difference method elsewhere. The results were compared with those of earlier workers for the restricted case of an isothermal coolant interface and a coolant in cross-flow but at constant temperature. They showed that there is better agreement between previous workers for the asymptotic values at very large interface areas. However, for realistic practical absorber lengths, the present results agree with one or other of the previous workers when they do not, in themselves, agree. It is likely therefore, that the present results can be considered reliable for realistic absorbers. It is shown that the absorption rate increases when the absorbent Lewis number is increased and decreases with an increase of heat of absorption. In terms of refri-

gerating capacity, these conclusions result in this factor increasing with the Lewis number and also with the heat of absorption. The effectiveness of the absorber was defined and the relationship between effectiveness, number of transfer units and coolant capacity ratio was presented. These results showed that the absorber effectiveness could be increased by increasing the coolant flow rate and/or increasing the contact area. It was indicated how a system could be optimized for maximum effectiveness at minimum cost by a careful balance of contact area against coolant flow rate.

*Acknowledgements*—This study was undertaken at King's College London and the authors are grateful for the facilities provided. One of the authors was financed by a grant from the Hariri Foundation and this was greatly appreciated.

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